Answers – Paper 1		
1	(a) ● $\begin{pmatrix}1\\1\\-1\end{pmatrix}-\begin{pmatrix}1\\0\\4\end{pmatrix}+\begin{pmatrix}8\\-6\\0\end{pmatrix}$ ● = $\begin{pmatrix}8\\-5\\-5\end{pmatrix}$ (b ● |r| = 5, ● $\begin{pmatrix}4/5\\-3/5\\0\end{pmatrix}$ or $\frac{1}{5}\begin{pmatrix}4\\-3\\0\end{pmatrix}$	2 2
2	●Midpoint is (5,1) ●M_{AB} = 4/6 = 2/3 ●M_{alt} = -3/2 ●Straight line is $y-1=-\frac{3}{2}(x-5)\rightarrow 2y=-3x+17$	4
3	● $f'(x)=\cos x+3$ ● $\cos\left(\frac{\pi}{3}\right)+3$ ● 3 ½	3
4	● $2^3+k(2^2)-4(2)-12=0$ ● k = 3	2
5	● *m* = tan150 = -tan30 ● $-\frac{1}{\sqrt{3}}$	2
6	● $4x^2+8x-5$ ● $4(x+1)^2$ ● $4(x+1)^2-9$	3
7	(a) ● $g^{-1}(x)=\frac{x-3}{2}$ (b) ● $f(2x+3)=\frac{1}{2x+3-4}$ ● $\frac{1}{2x+3-4}=\frac{1}{2x-1}$ (c) ● $x\neq$ ½	1 2 1
8	(a) ●RAT ● $\frac{\sqrt{100-2}}{10}=\frac{\sqrt{98}}{10}=\frac{7\sqrt{2}}{10}$ (b) ● $\sin(x+45)=\sin x\cos 45+\cos x\sin 45$. ● $\frac{7\sqrt{2}}{10}\times\frac{1}{\sqrt{2}}+\frac{\sqrt{2}}{10}\times\frac{1}{\sqrt{2}}$ ● $\frac{7}{10}\times\frac{1}{1}+\frac{1}{10}\times\frac{1}{1}=\frac{8}{10}$	2 3
9	(a) ● f'(x) = 0 ● $3x^2-12=0$ ● 3(x + 2)(x – 2) = 0 ● x= 2, x = -2 ● y = -14, y = 18 ● nature table or f''(x) ● maximum at (-2,18), minimum at (2, -14|) ●function is increasing when -2 < *x* and *x* > 2	7 2
10	● $x^2+(-2x+10)^2+2x-4(-2x+10)-15=0$ ● $5x^2-30x+45=0$ ● $5(x-3)(x-3)=0$ ● one point of contact at *x* = 3 proves tangency Or ● $b^2-4ac=0$ ● two real and equal roots proves tangency	4

11	• $f(x)=\int 2x-3\,dx$ • $f(x)=x^2+3x+C$ • y-intercept for both is (0,4) • $f(x)=x^2+3x+4$	4
12	(a) • $\frac{1}{2}(8-x^3)^{-1/2}\ldots$ • $\frac{1}{2}(8-x^3)^{-1/2}\times -3x^2$ • $-\frac{3x^2}{2(8-x^3)^{1/2}}$ (b) • make a connection with part (a) $(8-x^3)^{1/2}$ • full answer $-\frac{2}{3}(8-x^3)^{1/2}+C$	3 2
13	• $\boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\bullet\boldsymbol{b}+\boldsymbol{a}\bullet\boldsymbol{c}$ • $\boldsymbol{a}\bullet\boldsymbol{b}$ = 2x2xcos60 • $\boldsymbol{a}\bullet\boldsymbol{c}$ = 2x2xcos120 • $\boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c})$ = 2 + -2 = 0 • vectors are perpendicular when scalar product is zero, hence vector **a** is perpendicular to vector **b + c**	5
14	(a) • $\log_x y^3=\log_x y^2+2$, • $\log_x y^3-\log_x y^2=2\rightarrow\log_x\frac{y^3}{y^2}=2$ • $\log_x\frac{y^3}{y^2}=2\rightarrow\log_x y=2$ • $\log_x y=2\rightarrow y=x^2$ (b) • $y=(y-2)^2\rightarrow y=y^2-4y+4\rightarrow 0=y^2-5y+4$ • $0=y^2-5y+4\rightarrow 0=(y-4)(y-1)\rightarrow y=4\ and\ y=1$	4 2

Answers – Paper 2

1

(a) ● $\vec{RS} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$, $\vec{ST} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

● $\vec{RS} = 3\vec{ST}$, vector $\vec{RS}$ is a multiple of $\vec{ST}$ so these vectors are parallel

● vectors are parallel and share a common point S so R, S and T are collinear — 3

(b) ● S divides RT in the ratio 3:1 — 1

(c) ● $\vec{SF} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$, $\vec{ST} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ ● |SF| = $\sqrt{50}$ ● |ST| = $\sqrt{6}$

● $\vec{SF} \bullet \vec{ST} = 3\times1 + 4\times1 + -5\times2 = -3$

● angle FST = $\cos^{-1}\left(\frac{-3}{\sqrt{300}}\right) = 100^\circ$ — 5

Answer must reference $\vec{SF} \bullet \vec{ST}$ and not $a \bullet b$

2

Two sequences are defined by these recurrence relations

$U_{n+1} = 3U_n - 0.4,\ U_0 = 1$ and $V_{n+1} = 0.3V_n + 4,\ V_0 = 1$

(a) ● Vn has a limit as -1 < 0.3 < 1 — 1

(b) ● limit = $\frac{4}{1-0.3}$ ●limit is $\frac{40}{7}$ - only the **exact** value gets the mark — 2

(c) ● U_6 = 583.4 so U_7 = 1749.8 — 1

3

(a) ●Midpoint of QR is (2,4) ● M_{median} = 2 ●Equation is $y = 2x$ — 3

(b) ●Larger circle has a centre of (5,10)

●Larger circle has a radius of √20

●Points on line $y = 2x$ are (6,12), (7,14), (8,16) etc

●Distance between (5,10) and (7,14) is √20 hence C is (7,14)

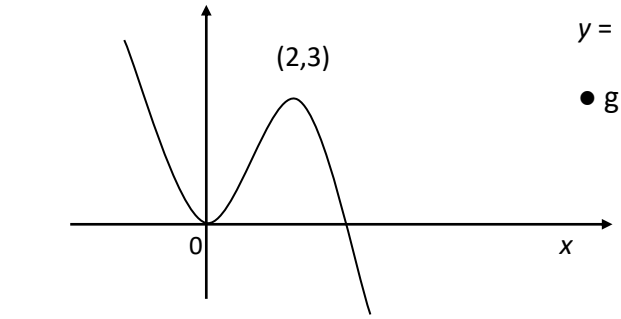
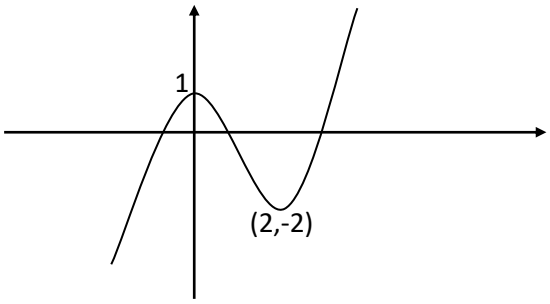
Or $7^2 + 14^2 - 10(7) - 20(14) + 105 = 0$

so (7,14) is a circumference point — 4

(c) ● Centre of smaller circle is (7,14) and radius is √5

● ● equation of smaller circle is $(x-7)^2 + (y-14)^2 = 5$

For $(x-7)^2 + (y-14)^2 = \left(\sqrt{5}\right)^2$ candidates lose one mark — 3

4	● $y = ax(x-4)$ ●$8 = a(2)(2-4) \rightarrow 8 = -4a$ ● $a = -2,\ b = 4$	3
5	(a) ● $k\cos(x+\alpha)^\circ = k\cos x\cos\alpha - k\sin x\sin\alpha$ ● $k\cos\alpha = 2, \quad k\sin\alpha = 5$ ● $\tan x = 5/2, \quad x = 68.2$ ● $k = \sqrt{25+4} = \sqrt{29}$ $\quad \sqrt{29}\cos(x+68.2)^\circ$ (b) range is $-\sqrt{29} \le y \le \sqrt{29}$ answer must be in terms of y	4 2
6	 $y = f(2x)$ ● graph scaled in x-direction  $y = 1 - f(2x)$ ● reflection in x-axis ●translation up 1 unit Candidates who reflect and translate the original graph still get 2 marks	1 2
7	(a) ● evidence of removing common factor $x(x^3-1)$ ● evidence of synthetic division using $x = 1$ and cubic factor (x^3-1) ● no remainder so $(x-1)$ is a factor of this polynomial ● quadratic factor (x^2+x+1) ● fully factorised $x(x-1)(x^2+x+1)$ (b) ●● $x = 1,\ x = 0$ ● quadratic factor (x^2+x+1) has no real root as $b^2-4ac = -3 < 0$ Candidates who stop at $x(x^3-1)$ can only be awarded 1 mark from part (a) and 2 marks from part (b)	5 3

8	• Sin2x = 2sinxcosx • 2sinxcosx – 2cos^{2}x = 0 • 2 cosx(sinx – cosx) = 0 • • cosx = 0 | sinx = cosx π/2, 3π/2 | π/4, 5π/4 Answers in degrees lose 1 mark	5
9	(a) • $\int_1^4 \frac{1}{x^2}dx$ • $\int_1^4 x^{-2}dx$ • $\left[\frac{x^{-1}}{-1}\right]_1^4 = \left[-\frac{1}{x}\right]_1^4$ • $\left(-\frac{1}{4}\right)-(-1)$ • Area = $\frac{3}{4}$ (b) • $\int_1^k \frac{1}{x^2}dx = \frac{3}{8}$ • $\left[-\frac{1}{x}\right]_1^k = \frac{3}{8}$ • $\left(-\frac{1}{k}\right)-(-1)=\frac{3}{8}$ • k= $\frac{8}{5}$	5 4
10	(a) • $9=10e^{-3k} \rightarrow \frac{9}{10}=e^{-3k}$ • $\log_e\left(\frac{9}{10}\right)=-3k$ • $k=\frac{\log_e\left(\frac{9}{10}\right)}{-3}=0.035$ (b) • $5=10e^{-0.035t} \rightarrow \frac{1}{2}=e^{-0.035t}$ • $t=\frac{\log_e\left(\frac{1}{2}\right)}{-0.035}=19.8$	3 2
11	(a) • Area of shed 3 = xy, y = 3/x • Area of Lawn A(x) = 3(4 + y) + 4x • Area of Lawn A(x) = 3(4 + 3/x) + 4x = 12 + 9/x + 4x = $12 + 4x + \frac{9}{x}$ (b) •Know to differentiate and equate to zero • $4 - \frac{9}{x^2}=0$ • $4x^2-9=0,\ (2x+3)(2x-3)=0,\ x=\pm 3/2$ • Nature table or use of the second derivative • A width or 1.5 metres minimises the area of the lawn	3 5
	END OF PAPER 2	

Log equations - exp half life - graphs of logs

Completing the square – inverse and composite functions – transformations of funct

Exact values – double angle – addition formula – wave function solving trig equations

Vector components – angle between vectors **- perpendicular vectors** – unit vectors – distributive law

Synthetic division – simultaneous equations - find the equation from the graph – using the discriminant

differentiation of composite functions – - rate of change - stationary points – equations of tangents – inc/dec functions – optimiation

Integrate a trig function – definite integrals – area between curves with difficult integral – differential equations

Parallel and perpendicular lines – m = tanx - medians, altitudes and perpendicular bisectors

General equation of a circle – tangents to circles – basic geom

Recurrence relations – problem solving with limits

New Prelim for Easter School